

# WAVE MODELING AT THE SERVICE OF SECURITY IN MARINE ENVIRONMENT

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## Summary

Wind-generated waves are ubiquitous on the ocean surface. Blowing winds excite oscillations of the water surface with length scales ranging from millimeters to about a kilometer and time scales from less than a second to tenths of a minute. The associated wave amplitudes vary from less than a centimeter to several meters and the high waves may affect the safety of humans at sea. The wish to protect humans and to prevent damage to ships and constructions, expedited the development of numerical wave models in the second half of the 20th century. Theoretical investigations of wave generation, nonlinear wave-wave interactions and wave dissipation in conjunction with experimental investigations led so far to three generations of ocean wave models. Today, third generation wave models are established world-wide. Numerical wave modeling is used in various areas, such as engineering, ship-routing, and real-time wave

prediction in connection with storm surge warning services. The growing interest in reliable real-time wave predictions calls for the development of a combined weather data and wave data assimilation scheme. Recent research corroborates the important role surface waves play in exchanging momentum, energy, gas and aerosols between the ocean and the atmosphere. Thus, in view of global climate changes, impact studies on potential wave climate changes and on potential changes in the marine ecosystem has opened a new area for wave modeling.

## **1. Introduction**

### **1.1 Classification of Waves**

Water waves represent a time-space varying phenomenon that obeys laws of geophysical fluid dynamics. Water waves are periodic motions of a density interface, and they propagate along the interface. These oscillations, either at the air-sea interface or at internal density interfaces below the sea surface, are transverse to the propagation direction.

This implies upward and downward motions of the interface about its equilibrium level, since the dominant restoring force of an elevated interface is the force of gravity. Note that this is different for sound waves, which oscillate in the longitudinal direction in alignment with the propagation direction.

Wind-driven waves at the sea surface are of major importance for humans at sea and at the seaside. The wind-driven surface waves represent small to medium scale waves. The periods of wind-driven surface waves vary between a fraction of a second and 30 s, their wavelengths vary between centimeters and 1000 m, and their heights (from trough to crest) vary between millimeters and 30 m. A single extreme wave height within a group of waves is known as a rogue or freak wave. Freak waves evolve momentarily in stormy weather and are extremely dangerous to ships.

The shortest water waves on the interface between water and air are the capillary waves. Capillary waves, which are restored by surface tension of water, have periods of less than one tenth of a second and wavelengths of about one centimeter. They play a role in surface wave generation from an initial flat surface.

Capillary waves coexist and interact with surface gravity waves. Knowledge of their properties is important for active microwave sensing, as microwave radars record signals backscattered by short waves with wavelengths of centimeters.

Examples of large-scale water waves are Tsunami, tides, Kelvin waves and Rossby waves. Tsunami are generated by underwater seismic activity and have periods in the range of 10 to 60 minutes. Tides are induced by gravitational forces of the Moon and the Sun exerted on the mass of the Earth. The dominant tidal periods are half-day and one-day long. Kelvin waves and planetary Rossby waves experience the effect of the Earth rotation (i.e., the Coriolis force) and are affected by lateral and bottom boundaries of the ocean basins. Interactions of the large-scale waves with the wind-driven waves can be neglected in usual conditions.

## 1.2. History of Wave Modeling

Wind-driven waves are ubiquitous on the ocean surface. The variable wind forcing, varying between weak winds and severe storms, induces surface waves of different amplitudes, periods, wavelengths and propagation directions. The thereby generated groups of waves transport energy along their propagation paths, and if propagation is linear their mass transport is nil. When waves reach coastal zones, the wave energy is transformed and dissipated by various processes. Since in extreme wave situations the safety of humans at sea and onshore can be severely threatened, the forecast of wave conditions had always been of concern.

The wish to forecast wave properties with wave models grew rapidly during the Second World War. Initially, a simple wave nomogram was developed to aid the planning of ship landings. Then, more sophisticated theoretical studies, and systematic laboratory and field measurements were conducted. Hence, numerical modeling of wind-driven waves developed progressively by reconciling theoretical knowledge with observational evidence. These efforts produced three generations of wave models. The distinction into three generations is connected with the treatment of the resonant nonlinear wave-wave interactions. These nonlinear interactions are the essential mechanism during wave growth, through which the peak frequency migrates to lower frequencies. The first generation models did not account for the nonlinear interactions, while the second generation models used a simplified parameterization of the nonlinear interactions. Then in third generation models, which are formulated for two-dimensional wave spectra, the nonlinear wave-wave interactions are explicitly computed.

Among the large number of measuring campaigns only the three most outstanding experiments in the field are briefly discussed here. The first experiment is the swell attenuation study in the Pacific presented by Snodgrass and others in 1966. The propagation of swell waves was traced over several thousands of kilometers. During their free propagation, the swell waves showed remarkable little attenuation. The second experiment, the Joint North Sea Project (JONSWAP), was conducted to measure the evolution of a wave spectrum in dependence of the wind vector, the fetch and other relevant parameters. The results, summarized by Hasselmann and others in 1973, yielded a parametric form of the frequency spectrum of wind-driven waves (JONSWAP spectrum) and empirical growth curves for the nondimensional wave energy and the nondimensional wave peak frequency. The nondimensionalization is obtained using elementary quantities, namely the wind stress forcing and the acceleration of gravity. The growth curves describe the exponential rates of the growth of nondimensional wave energy, and of the decrease of nondimensional peak frequency, both for growing fetch and growing duration of wind forcing. After sufficient long time of wind forcing, the speed of the dominant wave approaches the wind speed (taking a few hours for small fetches and small wind speeds until several days for long fetches and strong wind conditions). Then, the transfer of energy from the atmosphere eases off, and the wave growth reaches saturation. The third experiment, carried out in the Bight of Abaco and presented by Snyder and others in 1981, provided an empirical base for an appropriate wind input function in third generation wave models.

Today, third generation wave models are applied in different fields, such as for real-

time predictions of global and regional wave fields, for engineering purposes, for climate change studies, and for scientific research. The real-time predictions are useful for, e.g., the planning of optimal ship routes, and coastal warning services together with storm surge predictions. The interest of engineers is on the design of efficient ship structures, offshore constructions and coastal protection barriers, which can withstand the impact of waves. Analyses of long-term changes in the wave climate are essential for coastal protection measures. Aspects of scientific research are the refinement of the numerical modeling with respect to the manifold interaction processes at the interfaces between the water and the overlying air and the underlying bottom. Further research is undertaken to improve data assimilation schemes. Data assimilation is needed to exploit the real-time data from satellite-based remote-sensing instruments which provide a wealth of information on global wind fields and ocean wave fields.

### 1.3. Outline of the Chapter

In an international collaborative effort, in which numerous groups were engaged to develop an advanced and efficient wave model, the third generation wave model WAM was created. In the following, a few key points concerning the theoretical base of a third generation wave model will be addressed. For a broader elaboration on wave modeling the reader is asked to consult the annotated bibliography and the references therein.

Section 2 addresses the physical principles of free ocean surface waves. The wave field can be seen as an ensemble of monochromatic waves which randomly disturb the mean sea level. Useful definitions to describe the properties of a monochromatic wave, as well as the statistical properties of a field of waves are listed in the Glossary. Distributions of wave heights are usually described by either their probability distribution or their spectral power distribution (Section 2.1).

The mathematical-physical description of the kinematics and the dynamics of surface waves is based on two different approaches. The first approach uses the Navier-Stokes equation which is known in classical oceanography as the equation of motion in a viscous medium. This approach leads to the Laplace equation for waves in the domain of potential theory. The first-order solution of the Laplace equation, which is valid for small-amplitude waves, yields the dispersion relation for homogeneous media, elucidating the behavior of linear plane waves (Section 2.2).

The second approach of the mathematical-physical description employs Hamilton's principle of classical mechanics. The Hamilton principle is applied to an ensemble of linear waves which constitutes a wave group. The evolution of the properties of a wave group is then formulated in analogy to quantum mechanics for inhomogeneous media. In doing so, the ocean wave field is decomposed into different wave groups characterized by their wave numbers and their spectral action densities. The wave action density is the adiabatic invariant in an inhomogeneous system. Here, the inhomogeneous system is the ocean and the inhomogeneity is caused by varying ocean depth and currents. From the variational principle follows the radiative transfer equation of the spectral wave action density, which is also known as the transport equation of wave action in the domain of ray theory. In a homogeneous and time-invariant ocean the transport equation of the spectral action density reduces to the transport equation of

the spectral energy density (Section 2.3).

Usually, numerical wave models, describing the kinematics and dynamics of surface waves, are based on the radiative transport equation. One of the reasons is that the resonant nonlinear wave-wave interactions can be reconciled with the radiative transfer equation. The nonlinear interactions enter the transport equation as a forcing function in addition to the functions of wave generation and wave dissipation (Section 3). The nonlinear interactions are derived from theoretical considerations. They involve neither gain nor loss of the wave action, but a redistribution of wave action density among four resonant wave groups (Section 3.1). Wave generation is induced by the transfer of energy from the atmospheric flow to the sea surface (Section 3.2). Wave dissipation results from wave breaking at the surface and from friction at the sea floor (Section 3.3). The source functions of wave generation and dissipation are also theoretically founded, but require adjustment to empirical evidence.

Section 4 presents global applications of the third generation model WAM. The quality of global WAM predictions could be tested for the first time after polar orbiting satellites started to measure wave data by remote-sensing techniques. The agreement between monthly means of modeled and remote-sensed wave height data is reasonably good (Section 4.1). Real-time wave predictions, which are obtained by driving the wave model with actual wind fields from a synoptic weather model, are nowadays widely established. The synoptic wind forcing induces synoptic-scale variability in the wave data which frequently contain large deviations from the corresponding monthly mean (Section 4.2). Nowadays, wave forecasts a few days ahead are routinely carried out using assimilation methods involving coupled wave-atmosphere models (Section 4.3).

The closing section 5 gives a brief outlook on present wave modeling activities in the global and the regional perspective and in view of global climate change.

## **2. Physical Principles of Free Surface Waves**

### **2.1. Statistical and Spectral Characteristics**

Ocean surface waves appear as time-varying random elevations of the mean sea level. Such an appearance may arise from the superposition of many single sine waves with different periods, amplitudes and propagation directions. If the surface elevations are assumed to be locally homogeneous, stationary and Gaussian distributed, and the phases of the waves (i.e. the arguments of the sine functions) are evenly distributed, then the distribution of wave heights is completely described by the second statistical moment of an ensemble of randomly varying elevations. Based on the ergodic theorem, the second moment of an ensemble is equivalent to the variance of the elevation time series at a location.

The total energy  $E_{tot}$  per unit area of free sine waves is proportional to the variance of the surface elevations:

$$E_{tot} = \frac{1}{2} \rho g \langle \eta^2 \rangle , \quad (1)$$

where  $\langle \eta^2 \rangle$  is the variance of elevations  $\eta$ ,  $\rho$  the density of water and  $g$  the gravitational acceleration. Instead of using the total energy an effective height scale, i.e., the significant wave height  $H_s$

$$H_s = 4\sqrt{\langle \eta^2 \rangle} \quad (2)$$

is commonly used. The significant wave height can alternatively determined as the mean of the upper third of the height-sorted elevations. This follows from the Gaussian distribution of the elevations on the two-dimensional sea surface or from the narrow spectrum assumption. For the open ocean, the assumption of a Gaussian distribution is most of the time valid, as the space and time scales of generation and dissipation processes are usually much larger than the typical wavelengths and periods of the waves. Deviations from the Gaussian distribution and thus from linear wave theory occur in shoaling water with decreasing water depth, and especially in the surf zone. In the surf zone the waves become steep and nonlinear interactions induce wave breaking.

One advantage of the Gaussian assumption is that the statistical second moment of the surface elevations is directly related to the integral over all frequencies of the frequency spectrum

$$\langle \eta^2 \rangle = \int_0^\infty E(f) df \quad (3)$$

where  $E(f)$  is the spectral power density depending on frequency  $f$ . The spectral power density can be obtained according to the Wiener-Khinchine theorem by the Fourier transformation of the autocorrelation function of the elevation time series. The same transformation is applicable to a two-dimensional random wave field where the waves propagate in the horizontal plane. The spectral power density per unit area is then described by the two-dimensional wave number spectrum which can be equivalently expressed by the frequency-direction spectrum (see Section 4.2).

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Visit: <http://www.eolss.net/Eolss-sampleAllChapter.aspx>

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SWAN: <http://swan.ct.tudelft.nl/>

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Inventory of European wave data base:

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#### **Biographical Sketch**

**Dr. Eva Bauer**, studied Oceanography at the University in Kiel, Germany and received her Ph.D. in 1986. Thereafter, she worked at the Max-Planck Institute (MPI) for Meteorology in Hamburg on developing schemes for assimilating wave data into the spectral wave model WAM. She continued these studies at the Institute of Oceanography in Hamburg during her Assistant Professorship. Today, Eva Bauer is a senior scientist at the Institute for Climate Impact Research (PIK) in Potsdam in the field of Earth system climate models for paleoclimate research and climate change studies.